Time Value of Money and Capital Budgeting

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Overview

- ➢ Time Value of Money
  - – Compounding
  - – Annuity
    - • Perpetuity
    - • Growth
- ➢ Capital Budgeting
  - – Selection Criteria
    - • Financial
      - – Payback

- Discounted Payback
- Net Present Value
- Internal Rate of Return
- Benefit Cost Ratio
- Inflation
- Cost Concepts

- Multi Criteria Models
  - Unweighted 0 -1 factor model
  - Unweighted factor scoring
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Time Value of Money

➢One period investment

  – Project ends after one year
     Investment of Rs.1000
     Amount received after one year: Rs.1100

     Return on investment
          = (1100 − 1000) / 1000
          = 0.1 or 10%

– The amount to be received after one year may not be known with certainty

   If Rs.1100 is the expected amount after one year, then the expected return = 10%

Rs.1100 to be received one year hence. What is it worth today?

Discount rate depends upon

whether the amount to be received is known with certainty or not.

Suppose the discount rate is 8%

Then Present Value, PV, of
 future cash flow of 1100 is
(1100) / (1.08)= 1018.52

Invest in the project since PV of
Future cash flow > Cost of
 investment

Compounding

❖ Suppose Rs.100 invested earns 8% compounded annually for two years

Then at the end of one year, the investment will be worth
100 (1.08) = 108

❖ At the end of two years, the investment will be worth

$$108\ (1.08) = 100\ (1.08)\ (1.08)$$
$$= 116.64$$

Or $\quad = 100\ (1.08)^2$
$$= 100\ (1.1664)$$
$$= 116.64$$

❖ Semiannual Compounding

Suppose Rs.100 invested earns 4% compounded semi-annually.

At the end of one year, the investment will be worth

100 (1.04) (1.04) = 100 (1.0816)

$$= 108.16$$

Note: The annual return is 8.16% which is greater than 8%

❖ **Quarterly Compounding**

Suppose Rs.100 invested earns 2% compounded quarterly.

At the end of one year, the investment will be worth

$100 (1.02)^4 = 100 (1.0824) = 108.24$

Note: The annual return is 8.24% which is greater than 8.16%

❖ Continuous Compounding

   Suppose r is the stated annual rate compounded continuously.

   Then Rs.100 invested for T years with continuous compounding will, at the end of T years, be worth $100\,e^{rT}$ where e = 2.718

   Suppose r = 8% and T= 1 year, then $100\,e^{0.08(1)}$ = 108.33 or a return of 8.33%

   If T = 2, $100\,e^{0.08(2)}$ = 117.35

❖ Power of compounding

Suppose Rs.100 invested earns 10% compounded annually for say 50 years.

At the end of 50 years, the investment will be worth

$100 (1.1)^{50} = 11,739.09$

❖ Annuity into Perpetuity

An amount A is received annually one year from now into perpetuity.

Present Value, PV = $[A / (1 + r)] + [A / (1 + r)^2] + [A / (1 + r)^3]$ +…………

= A / r

Suppose A= 100; r= 0.1

PV = $100 / 0.1 = 1000$

❖ Annuity for T years

An amount A is received annually one year from now for T years.

$$PV = [A / (1 + r)] + [A / (1 + r)^2]$$
$$+ \ldots\ldots + [A / (1 + r)^T]$$
$$= A [(1 + r)^T - 1] / [(1 + r)^T (r)]$$

Suppose A = 100; r = 0.1; T = 8
$$PV = 100 [(1.1)^8 - 1] / [(1.1)^8 (0.1)]$$
$$= 533.49$$

❖ Annuity Growing into Perpetuity

An amount A is received one year from now.

From the end of second year onwards the amount received at the end of a year is (1 + g) times the amount received at the end of the previous year.

Assume the growth factor g is less than the discount rate r.

$$PV = [A / (1 + r)] + [A (1+g) / (1+r)^2]$$
$$+ [A (1+g)^2 / (1+r)^3] + \ldots \ldots = A / (r-g)$$

Suppose A = 100; r = 0.1 and
 g = 0.05

$$PV = 100 / (0.1 - 0.05) = 100 / 0.05$$
$$= 2000$$

Multi-period investment

❖ Investment of Rs.1000

Amount received after one year = Rs.600

Amount received after two years = Rs.800

Suppose the discount rate is 10%.

Present value (PV) of future cash flows

$=(600) / (1.10) + (800) / (1.10)^2$

$= 1206.6$

Net Present Value = PV of future cash flows − Investment cost

$= 1206.6 − 1000$

$= 206.6$

Capital Budgeting

❖ Projects classified into specific areas / buckets

❖ Projects in different buckets may be evaluated using different criteria

❖ Within each bucket rank order projects using the same criteria

❖ Buckets may be further sub-divided

Core Technology Development	Rs. Crores	New Product Development	Rs. Crores
	10.00		12.00
Project 4	4.60	Project 3	4.50
Project 2	5.40	Project 1	2.70
		Project 6	4.80

❖ Selection Criteria

  ➢ Financial

  ➢ Non-Financial

❖ Financial Models

  ✓ Single criteria

  ✓ Easy to use

❖ Payback period

  ➢ Requires estimation of cash flows

➢ Example

Initial Cost = 100,000

Annual savings or incremental net cash in flow = 25,000

Payback = 100,000 / 25,000 = 4

Another Example:

Year	1	2	3
Incremental Net Cash Inflow Rs.	30,000	24,000	20,000

Pay back period = 1 + 20,000 / 24,000
= 1 5/6 years

Discounted Payback:

Discount Rate = 10%
Initial investment = 50,000

Discounted Cash flows :
 Year 1: 30,000 / 1.1 = 27,272.13 ;
  Year 2: 24,000 / (1.1)^2 = 19,834.71 ;
  Year 3: 20,000 / (1.1)^3 =15,026.30
Discounted Payback
= 2 + [50,000 – (27,272.73 +
        19,834.71)] / 15,026.3
= 2.1925

❖Accept the project if payback / discounted payback period is less than a specified value

❖Incremental Net Cash flows after payback/ discounted payback period are ignored

❖May be used as a screening device if long payback/discounted payback is not acceptable

❖Estimation of incremental net cash flows over a long time horizon may be highly uncertain / difficult

❖ Net Present value

➤ Requires estimation of cash flows and cost of capital

I_o : Initial investment (negative because it is an outflow)

F_t : Incremental Net Cash flow for period t (negative if net cash flow is an outflow)

k : required rate of return

n : Life of the output of the project

- Note: F_n will include the salvage value of the assets created by the project
- Project NPV

$$= I_0 + \sum_{t=1}^{n} F_t / (1 + k)^t$$

k is typically the weighted average cost of capital (WACC)

Alternatively k may depend on the riskiness of the project

- ➢ Previous Example
- ➢ Suppose k= 10%
- ➢ NPV = - 50,000 + 30,000 / 1.1

  + 24,000 / $(1.1)^2$ + 20,000 / $(1.1)^3$

  = - 50,000 + 27,273 + 19,835

      + 15,206

      = 12,134
- ➢ If NPV > 0, accept the project
- ➢ Ignores size of initial investment

➤ Internal Rate of Return

I_o : Initial investment (negative because it is an outflow)

F_t : Incremental Net Cash flow for period t (negative if net cash flow is an outflow)

k : Internal rate of return

n : Life of the output of the project

F_n : includes the salvage value of the assets created by the project

➢ Find k such that

$$I_0 + \sum_{t=1}^{n} F_t / (1+k)^t = 0$$

k is found by trial and error

- Accept project if   k > hurdle rate

- Requires estimation of cash flows and hurdle rate

- Possibility of multiple internal rates of return if some of the future cash flows, F_t, are negative i.e. are outflows

❖ Previous Example
❖ Find k so that

  - 50,000 + 30,000 / (1+k)
 + 24,000 / (1+k)^2 + 20,000 / (1+k)^3
  = 0

By trial and error, IRR is approximately
24.42%

❖ **Benefit Cost Ratio**

✓ **Also called Profitability Index**

✓ **= (present value of all future incremental cash flows) / (Initial Investment)**

✓ **Accept project if this ratio is > 1**

✓ Previous Example

   Present Value of Incremental cash flows

= 30,000 / 1.1 + 24,000 / $(1.1)^2$
   + 20,000 / $(1.1)^3$ = 62,134

✓ Benefit Cost Ratio = 62,134 / 50,000
                            = 1.2427

❖ Mutually exclusive projects

❖ Only one of the projects A or B can be undertaken

- Different ways of extending the capacity of warehouse

For payback / discounted payback criterion accept the project with least payback / discounted payback period if it is less than some specified value

For NPV, accept the project with highest NPV if it is greater than 0

- For IRR, accept the project wit the highest IRR if it is greater than the hurdle rate

- For Benefit Cost Ratio, accept the project with the highest Benefit Cost Ratio if it is greater than 1

- Contingent projects
  - ✓ Project B can be accepted only if Project A is accepted
  - ✓ Now we have two mutually exclusive cases

Project A only or Project A + Project B

- ❖ Resource constrained projects
  - ➤ NPV criterion
  - ➤ Optimization model
    - ✓ Integer programming

Inflation

- Suppose nominal interest rate = 10% per annum.

  Suppose inflation is estimated to be 7% per annum.

  The real interest rate is approximately

  = Nominal interest rate − inflation rate

  = 10 − 7 = 3%

The exact real rate is calculated as follows:

Rs.1 invested will at the end of the year grow to (1 + nominal interest rate)

But purchasing power of this amount is only [(1 + nominal rate)] / (1 + inflation rate)

So the real gain or real rate of interest = [(1 + nominal rate) / (1 + inflation rate)] − 1

The exact real interest rate

= (1 + 0.1) / (1 + 0.07) − 1

= (1.1) / (1.07) − 1 = 0.02804 or 2.804%

Note: The approximation is good only if the nominal rate and inflation rate are small.

Suppose nominal interest rate = 100% and inflation rate = 80%

Approximate real interest rate
= 100 − 80 = 20%

The exact real interest rate
= (1 + 1) / (1 + 0.8) − 1
= (2 / 1.8) − 1
= 0.1111   or   11.11%

Note: Nominal cash flows should be discounted at nominal rates.

Real cash flows should be discounted at real rates.

Previous example:

Initial cost = -50,000

Nominal discount rate = 10%

Estimated inflation rate = 7%

Real rate = (1 + 0.1) / (1 +0.07) − 1

=0.02804 or 2.804%

Year	1	2	3
Incremental Net Cash Inflow (Nominal)	30,000	24,000	20,000

Incremental Net Cash Inflow (Real):
Year 1: $30,000 / 1.07 = 28,037.38$
Year 2: $24,000 / (1.07)^2 = 20,962.53$
Year 3: $20,000 / (1.07)^3 = 16,325.96$

Discounting at real rate of 2.804%

Present Value of Cash flows:

Year 1: 28,037.38 / 1.02804 = 27,272.66

Year 2: 20,962.53 / $(1.02804)^2$

$= 19,834.61$

Year 3: 16,325.96 / $(1.02804)^3$

$= 15,026.18$

Net Present Value

$= -50,000 + 27,272.66 + 19,834.61$
$+ 15,026.18 = 12,133.45 \approx 12,134$

Cost Concepts

- Depreciation
  - Initial investment of Rs.1,00,000 is depreciated over 5 years say using straight line depreciation.

  For accounting purpose, (1,00,000) / 5 = 20,000 is treated as depreciation.

  For capital budgeting, the entire amount of Rs.1,00,000 is initial investment cost.

- Sunk Costs
  - You have hired a consultant and paid Rs.10,00,000 last year to identify strategic growth plan for your organisation.

  Now you want to evaluate projects that have been identified.

  The amount paid to consultant is a sunk cost.

- Opportunity Cost
  - You have an empty warehouse which could be sold. Now you are considering a project which can utilise the warehouse space.

  The selling price of the warehouse is an opportunity cost for the project.

❖ Disadvantages of Financial Models

  ➢ Ignores all non-monetary factors

  ➢ Biased towards short run

  ➢ Difficult to forecast incremental cash flows

  ➢ Sensitive to errors in estimating incremental cash flows during the early years

➢ Difficult to estimate the life of the output of the project and th salvage value of the assets created by th project

➢ The effect of changes (or errors) in incremental cash flows and / or cost of capital is not easy to comprehend

Multi-criteria Models

❖Uses several criteria

  ❖Vary in complexity

  ❖Different information requirements

  ❖Relevant factors or criteria are selected

  ❖Unweighted 0-1 factor model

    ❖For each factor, the rater / decision maker scores the project in terms of qualifies or not qualifies

❖ The number of qualifying factors is calculated
  If this number is sufficient, the project is selected

❖ Disadvantage is that all factors are given equal importance

❖ No gradation of the degree to which the factor is satisfied

Project _____   Date _____
Rater _____

	Qualifies	Does Not Qualify
	X	
No increase in energy requirements	X	
Potential market size, dollars	X	
Potential market share, percent	X	
No new facility required		X
No new technical expertise required	X	
No decrease in quality of final product		X
Ability to manage project with current personnel	X	
No requirement for reorganization	X	
Impact on work force safety	X	
Impact on environmental standards		
Profitability	X	
Rate of return more than 15% after tax	X	
Estimated annual profits more than $250,000	X	
Time to break-even less than 3 years		X
Need for external consultants		X
Consistency with current line of business		
Inpact on company image		
With customers	X	
With our industry		X
Totals	12	5

Figure 2-2   Sample project evaluation form.

❖Unweighted Factor Scoring Model

➢A numerical score is given to the degree to which each criterion is met

➢Typically a five point scale is used

➢For each criteria, five ranges of performance level are identified

✓Typically the ranges are chosen so that form past experience, roughly equal number of projects will fall in each range

✓It is not necessary to have equal ranges

✓ Example

Score	NPV
5	Above 1,000,000
4	750,000 to 1,000,000
3	400,00 to 750,00
2	100,000 to 400,000
1	Less than 100,000

Score	Ability to manage project with current personnel
5	Very easy and definitely possible
4	Easy without much effort
3	Possible with extra effort
2	Difficult
1	Very difficult

❖ For each project, the scores for the different criteria are added to get the total score

❖ The projects are ranked based on the total score

❖ Assumes all criteria are equally important

❖ Weighted Scoring Model

s_{ij} : Score of the i [th] project on j [th]

criteria

w_j : Weight for j [th] criterion

n : Number of factors or criteria

R_i : Total score for i [th] project

$$R_i = \sum_{j=1}^{n} w_j \, s_{ij}$$

➢Desirable to normalize the weights so that

$$\sum_{j=1}^{n} w_j = 1$$

➢This would give an idea about the relative importance of the criteria

➢How to calculate the weights / relative importance of criteria
  ✓ Internal team
  ✓ Analytic Hierarchy Process

Thank You